The Chinese Numeration System and Place Value

Ichi, ni, san . . . *Otu, abuo, ato* . . . *Eins, zwei, drei* . . . Have you heard these words before? What might they mean? In case you are not familiar with these words, they mean "One, two, three . . ." in Japanese, Igbo, and German, respectively.

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Two of the most fundamental topics in mathematics that children must learn are counting and the numeration system. Central to the Number and Operations Standard of NCTM's *Principles and Standards* is the development of number sense, which includes the ability to understand the base-ten number system (NCTM 2000). Reys (1998) explains that place value is the foundation of our numeration system. We use place value to read, represent, and operate on numbers and also to identify patterns, build number sense, and understand different numeration systems. This article presents the Chinese numeration system as an additional representation of numbers and place value for teachers to consider in their instruction. Sample learning activities using the Chinese numeration system for elementary students are provided.

The Chinese Numeration System Explained

Virtually every civilization and culture developed the concept of numbers and the formulation of a counting process (Eves 1990). Cultural differences led to the creation of numeration systems suited to the unique needs and purposes of those who used them (Cathcart et al. 2000). These needs ranged from tallying the number of animals kept to developing calendars. Many cultures created culturally specific numeration systems that included their own symbols, rules, and values. One of these is the Chinese numeration system.

The Chinese numeration system is a decimal (base-ten) system, unlike other systems such as the Babylonian (sexagesimal or base-sixty) or the Mayan (vigesimal or base-twenty). In the decimal system, counting is based on ten numerals, symbols are used to represent the numerals, and successive place values—such as 10; 100; 1,000; and 10,000—are powers of ten (Baumgart et al. 1989). **Figure 1** shows the characters used in the Chinese numeration system.

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What is unique about this numeration system? The written form indicates the number of units in each base, together with that base. For example, the number 72 is actually written as *seven ten(s) and two* (七十二). In contrast, in the Hindu-Arabic system, the written numerals indicate digits that are positioned accordingly, and each digit is a representation of a multiple of some power of the chosen base (Eves 1990). Consider the number 564. The “5” has a value of $5(10^2)$, or 500; “6” has a value of $6(10^1)$, or 60; and “4” has a value of $4(10^0)$, or 4. In the Hindu-Arabic system, the base (10) is implied. Written in Chinese characters, 564 is 五百六十四 (5; 100; 6; 10; and 4), which literally means *five hundred(s), six ten(s), and four.*

In the Chinese numeration system, corresponding characters exist for 0–9 and for the multiples of 10, that is, 10; 100; 1,000; and so on. A number can easily be rewritten from Hindu-Arabic to Chinese by writing it in its expanded form, which gives the actual value of each digit. For primary students, it is suggested that a number be expanded as a sum of its parts (Cathcart et al. 2000). An example is 745 = 700 + 40 + 5, or 7 hundreds + 4 tens + 5. For intermediate students, the distributive property can be used: $745 = (7 \times 100) + (4 \times 10) + 5$ or $(7 \times 10^2) + (4 \times 10^1) + (5 \times 10^0)$.

Consider the number 15 and its expanded form, 10 + 5. Using the table in **figure 1,** 10 is represented as 十 and 5 as 五. Therefore, 15 written in Chinese will appear as 十五. Another example is 79. In expanded form, 79 = 70 + 9. In Chinese, 70 is written as 七十 and 9 as 九, and the combination of the two is 七十九. The principle is the same for greater numbers: 193 = 100 + 90 + 3. Putting the corresponding characters together for these numbers, 193 is 一百九十三. These concepts also can be extended to numbers in the thousands and ten thousands. If written properly, 2,745 in Chinese will appear as 二千七百四十五 *(two thousand[s], seven hundred[s], four ten[s], and five)* and 39,681 as 三萬九千六百八十一 *(three ten-thousand[s], nine thousand[s], six hundred[s], eight ten[s], and one).*

FIGURE 1

The Chinese numeration system

0	zero	零	10	ten	十
1	one	一	100	one hundred	百
2	two	二	1,000	one thousand	千
3	three	三	10,000	ten thousand	萬
4	four	四	100,000	one hundred thousand	十萬
5	five	五	1,000,000	one million	百萬
6	six	六			
7	seven	七			
8	eight	八			
9	nine	九			

A natural question that will arise is, what if there is a zero digit? If the zero digit is at the end of the number, there is no issue. 5,300 will be written as 5,000 + 300; in Chinese, this is 五千三百 *(five thousand[s], three hundred[s])*. The more interesting scenario is when the zero digit lies in the middle of the number. Consider 6,028 and its expanded form of 6,000 + 20 + 8. In a case such as this, it is recommended that zero be considered a part of the expanded notation. Therefore, 6,028 is better written as 6,000 + 0 + 20 + 8. Using the Chinese characters correspondingly, 6,028 now looks like 六千零二十八. Try writing 90,999 in Chinese. You should get 九萬零九百九十九. For sample classroom activities, see **figures 2–4.**

The Field Test

The activity in **figure 2** was tried in a fourth-grade class in Rio Vista Elementary School in Pico Rivera, California. The students were Hispanic, and English was not the primary language at their homes. This lesson was chosen to serve as an extension of place value and to expose the students to a different way to name numbers and express place value than they would find in either Spanish or English. Before doing the activity in **figure 2,** the students were given base-ten blocks to use with place-value activities. This served as a review of base-ten concepts, expanded form, and groupings that represent numbers and acted as a bridge between concrete and symbolic representations of numbers. The author and the teacher collaborated in introducing and demonstrating the Chinese numeration system to the students. After the lesson was conducted, students then were asked to complete Activity 1 and write about their understanding of place value.

Two samples of student work are shown in **figure 5.** The first sample indicates that the student could translate from Hindu-Arabic to Chinese readily and had a procedural knowledge of the value of a digit, its place value, and its symbolic representation. In the second sample, the student wrote the Chinese representation for each digit without writing the place value, or simply left them blank. When asked why they did this, students who made this mistake said that they translated exactly the digits from Hindu-Arabic to Chinese and did not consider the place values of the digits. This served as a flag for the teacher to inquire into the understanding of these students. After doing the activity, some students still seemed to experience difficulty with place value and expanded forms of numbers. The teacher then attempted to clear up the confusion by modeling the place values, using concrete representations such as base-ten blocks while writing the expanded notations for each number.

FIGURE 2

Activity 1: Complete the table below.

Number	Expanded Form	Chinese System
(a) 398	300 + 90 + 8	
(b) 751	700 + 50 + 1	
(c) 576		
(d) 307	300 + 0 + 7	
(e)	4,000 + 500 + 80 + 2	
(f) 7,500		
(g) 80,427		
(h) 38,074		

Benefits of the Lesson

Several benefits of this instructional approach were noted. First, the activities reinforced the students' knowledge of place value. According to the Number and Operations Standard in *Principles and Standards for School Mathematics*, instructional programs should enable students to understand numbers, ways of representing numbers, relationships among numbers, and number systems (NCTM 2000). These activities were aligned with the Standard. Furthermore, students first wrote the given

FIGURE 3

Activity 2: Use Chinese characters to write your answers.

Questions	Answers
(a) What is the value of "8" in 4,834?	
(b) What year were you born?	
(c) What year did Arizona become a state?	
(d) What is the weight in ounces of an object weighing 1,032 pounds?	
(e) How many times does your heart beat in three minutes?	

FIGURE 4

Activity 3: Answer the following questions.

1. Your classmate wrote 4,732 as 四千七十三百二.
Is this correct or incorrect? Explain.

2. Write 378 in Chinese and how you will explain to a friend what the characters mean.

3. How do you think "ten million" is written in Chinese? Try it.

numbers in the expanded form before using Chinese characters, demonstrating their understanding of the relationships between the base-ten place values. The students also reviewed the relationship between the value of each digit and the total value of the numbers as recommended by Heddens and Speer (1997). When asked how to write numbers in Chinese, students said, "First you write the symbol for that number and then write the symbol for its value."

A second benefit of the activity was that it created a foundation for students' algebraic thinking. The Chinese numeration system shows the number of units in each base, along with that particular base. This thinking involves the concepts of coefficients and "unknowns" (variables); that is, the number of units in each base represents the coefficient, and the base is the variable.

Finally, this type of activity emphasizes the fact that mathematics permeates all cultures and societies. *Principles and Standards for School Mathematics* recognizes that mathematics is part of cultural heritage. It states further that mathematics is considered to be one of the greatest cultural and intellectual achievements of humankind, and an appreciation and understanding of that achievement must be developed (NCTM 2000).

An earlier study conducted by the author concluded that when a lesson is presented through a cultural or historical connection, students see more of the value and use of mathematics in the world outside the classroom and gain a better appreciation for the mathematics that arises (Uy 1996). A multicultural approach to teaching mathematics can enable students to take pride in the accomplishments of their own people (Zaslavsky 1991).

Conclusion

The Chinese numeration system is one of many numeration systems that can reinforce a student's conceptual knowledge of place value. Through the use of Chinese numeration representations, students reviewed the base-ten place values in other contexts. Introducing a different cultural perspective toward a mathematical concept shows students that there are other ways to learn mathematics. The sample activities presented use a multicultural approach. You are invited to try them and offer feedback about students' mathematics learning through them.

Some other suggested activities include examining the origin and history of the Chinese numeration system, researching other numeration systems, comparing numeration systems, and reading numbers in different languages. Another suggestion is to continue to have students write their reactions to or reflections about the lesson. The students' reactions and reflections will give teachers insights into students' understanding of and appreciation for base-ten and other numeration systems. During the study, one student noted that she finally understood place value after being introduced to the Chinese numeration system.

Students in today's mathematics classes have diverse backgrounds, and we teachers should employ techniques and methods that will accommodate their needs. Maintaining meaningful contexts for our lessons is essential. The use of the Chinese numeration system to enrich students' understanding of place value is only one example of the many cultural numeration systems that you can investigate and share with your students.

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FIGURE 5

Sample student work

Number	Expanded Form	Chinese System
(a) 398	300 + 90 + 8	三百九十八
(b) 751	700 + 50 + 1	七百五十一
(c) 576	500 + 70 + 6	五百七十六
(d) 307	300 + 0 + 7	三百零七
(e) 4582	4,000 + 500 + 80 + 2	四千五百八十二

Number	Expanded Form	Chinese System
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(b) 751	700 + 50 + 1	七五一
(c) 576	500 + 70 + 6	五七六
(d) 307	300 + 0 + 7	三零七
(e) 4582	4,000 + 500 + 80 + 2	三零

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