Culture in the

Enriching Numeration and Number Operations

Students hear, "Pretend that you live on an island. You have no contact with others. You do not know words or written symbols for numbers. Your task is to develop a numeration system—a method that would allow you to count coconuts or shells, for example." This task was the first one given to fourth graders in an attempt to embed multiculturalism in their mathematics curriculum. This article describes the content of the project and the students' reactions.

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Beginning the Project

For the task of inventing a numeration system for the island inhabitants, the students worked in groups in various ways. For example, one group discussed ideas together while another worked individually with the intent to vote on the best system. One student's symbols are shown in **figure 1.** As the students discussed their ideas, a window was opened for examining their reasoning.

One group considered the idea of using the first letter of English words for numerals, but they then had to deal with words that have the same first letter, such as *four* and *five*. One student asked, "How would we know what *F* looks like?" They were truly intrigued by the mental imagery of their problem situation. One student challenged another to write his name as he would write it on the island. Another created a symbol for addition, coming up with his own extensions to the exploration. The students enjoyed the creative expression offered by the activity and asked, "Are we going to make up a new language?" My response was that we would not make up an entirely new language, but later, we would take a look at some words for numerals in other languages. This introduction led into our work with numeration systems and algorithms from other cultures.

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The author expresses appreciation to Mary Elizabeth Wilson-Patton, who served as coordinator of the English as a Second Language Program at Berry College from 1998 to 1999. Her suggestion to develop an article gave birth to this project. Thanks are also extended to the Berry College Elementary School faculty and administrators, who welcome the teacher education faculty with open arms. Finally, the author is especially grateful to the delightful fourth graders who participated in this project.

Egyptian Numerals

First we looked at the written symbols of the Egyptian numeration system. Interestingly enough, the fourth-grade "inhabitants" of one island had created a positional numeration system, which opened a discussion of the differences between additive and positional systems.

Introducing students to Egyptian numerals also added meaning to our own decimal system of numeration. **Table 1** shows that both the Hindu-Arabic system and the Egyptian numeration system use a base of ten. The need to create a new symbol for each group of ten of the previous value shows students the features and limitations of a numeration system that does not have place value.

Curriculum:

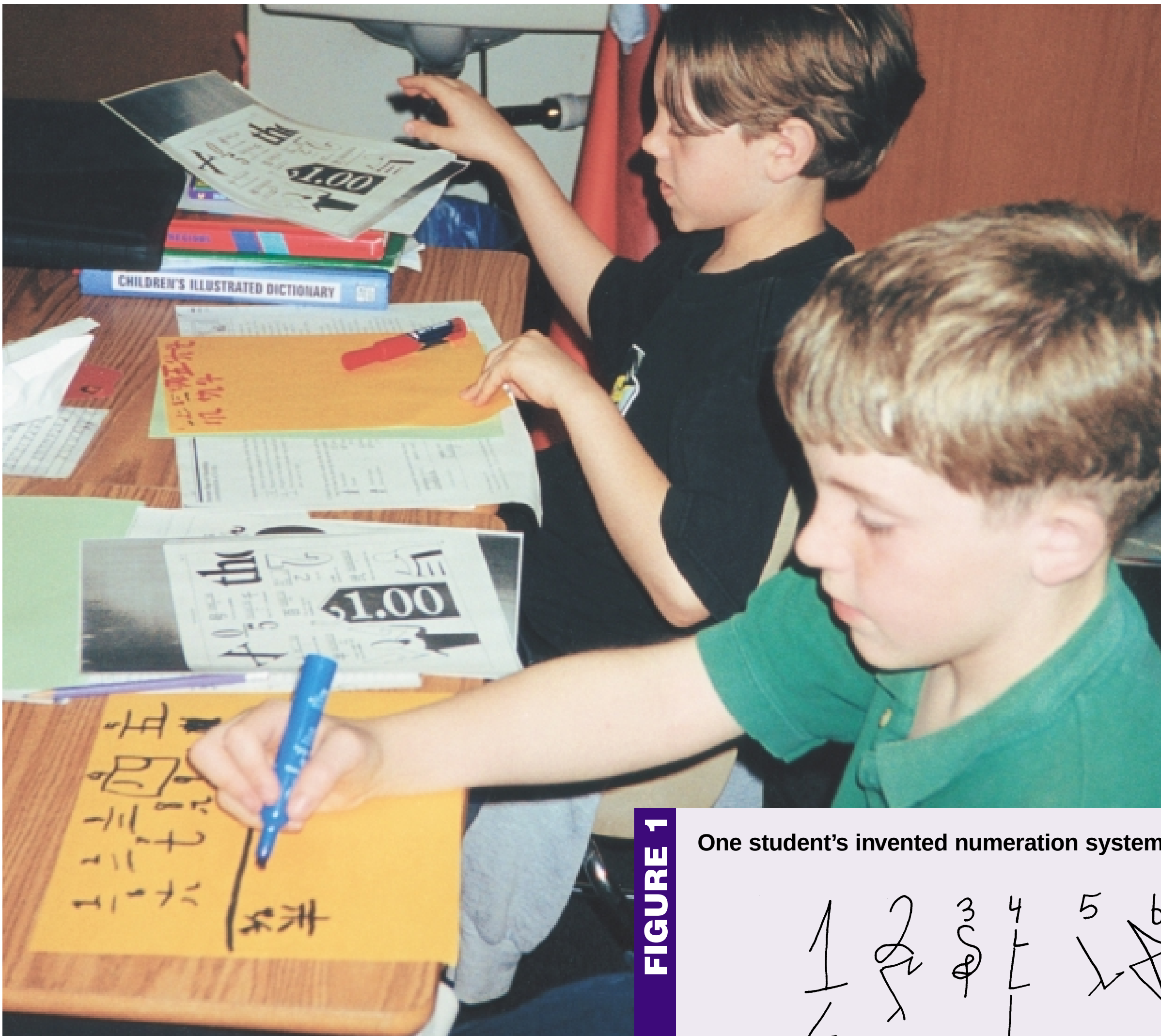
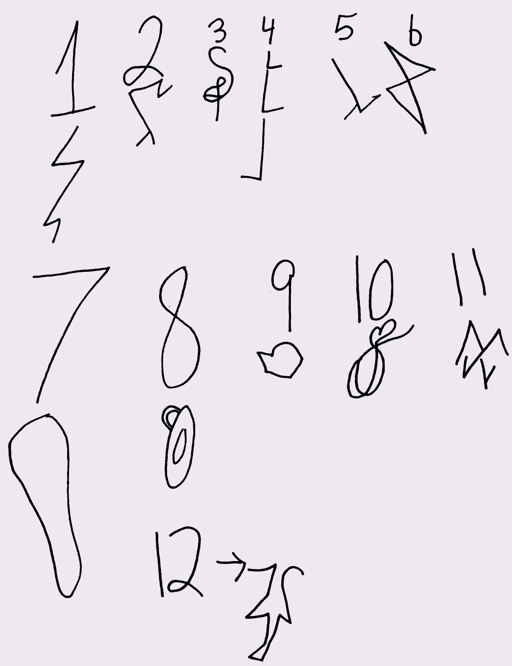


FIGURE 1

One student's invented numeration system



They also come to understand the need for zero in our own system while recognizing that no symbol is needed for zero in an additive system, such as the Egyptian.

The students thoroughly enjoyed the process of changing decimal-system numerals to Egyptian numerals, and vice versa; for example,

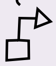
23 = ∩∩||| and 𓍢∩∩∩∩|| = 142.

For this activity, I gave students whiteboards and dry-erase markers to facilitate sharing their work. After practicing writing numerals, we moved on to addition and subtraction with Egyptian numerals.

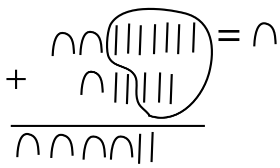
Egyptian numerals helped students explore the

TABLE 1

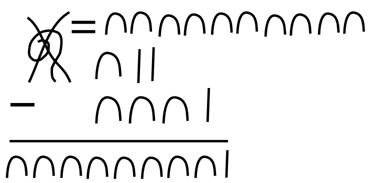
A comparison of Hindu-Arabic and Egyptian numerals

Hindu-Arabic Numeral	Egyptian Numeral	Name of Egyptian Numeral
1		Vertical staff
10		Heelbone
100		Scroll or coiled rope
1,000		Lotus flower
10,000		Pointing finger
100,000		Pollywog, burbot fish, or tadpole
1,000,000		Astonished man

meaning of *regrouping*. For example, in the following addition problem, ten vertical staffs are exchanged for one heelbone:



Similarly, in the following subtraction problem, a scroll is replaced with ten heelbones:



Although teachers would like to assume that fourth graders understand place value and regrouping, their understanding may be incomplete or they may be manipulating numbers by rote. Exploring place value and regrouping through Egyptian numeration is a fresh, new way to deepen students' understanding.

Algorithms for Subtraction: Left to Right and Equal Addition

After doing addition and subtraction using Egyptian numerals, the students were given base-ten blocks and asked to illustrate sample problems. Given a problem such as 34 – 18, for example, the students were asked whether the problem could be solved by taking away the one "long" block (group of ten) first. The students carried out this procedure with base-ten blocks, and then I introduced a way to show the steps of this left-to-right algorithm.

$$\begin{array}{r} 34 \\ \underline{-18} \\ 2 \end{array}$$

One long is taken away, leaving two longs. Then one of those longs is exchanged for ten 1's:

$$\begin{array}{r} {\scriptstyle 1} \\ 34 \\ \underline{-18} \\ \not{2} \\ 1 \end{array}$$

Then the eight singles are subtracted from the fourteen singles:

$$\begin{array}{r} {\scriptstyle 1} \\ 34 \\ \underline{-18} \\ \not{2} \\ 16 \end{array}$$

Students understood that this algorithm produced the same result and is an acceptable way to perform subtraction. I then introduced another algorithm for subtraction, the equal-addition method.

To solve the same problem, 34 – 18, using equal addition, an equivalent problem is created by adding the same number to the minuend and subtrahend. Ten 1's are added to the minuend, and one group of 10 is added to the subtrahend:

$$\begin{array}{c} 34 + 10 \text{ ones} = 3 \text{ tens } 14 \text{ ones} \\ \underline{18 + 1 \text{ ten} = 2 \text{ tens } 8 \text{ ones}} \end{array}$$

Students checked their results by doing the problem "their" way, that is, using the standard right-to-left algorithm with regrouping, or "borrowing," one of the three groups of 10 into ten 1's. Again, they were convinced that this new method would also produce correct answers.

The students were undoubtedly getting the message that mathematics can be done in more than one way. To check their responses, I gave them an open-ended writing prompt: "Think about what we have talked about during mathematics time for the past few Wednesdays. Write about what you have learned." Josh wrote, "Because where we will learn more than 1 way to do math and we will not take at grented that there is only 1 way to do math." Will's reflection a few weeks later also indicated that this enrichment project taught him that other countries do not use the same methods that we do in the United States: “I learned that the other countries don’t use the same method of math and I will probably remember the lattice method” (see the next section).

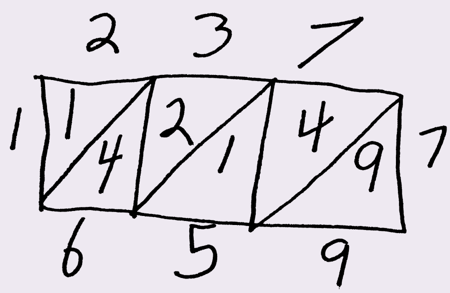
I also wanted the students to understand that these other algorithms are actually in current use. I explained that when children are not “taught” the procedures for addition and subtraction, they will naturally do the operations using the left-to-right algorithm (Kamii 1987). I also mentioned that I knew some adults who were schooled in other countries and had been taught the equal-addition method, also called the “New Zealand method.”

Lattice Method for Multiplication

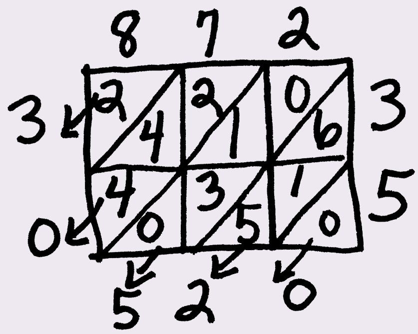
Will’s use of the lattice method to multiply 237 by 7 is shown in **figure 2a.** For each partial product, the tens digit is written in the upper-left corner and the units digit is written in the lower right. Partial products are then added along diagonals running from the lower left to the upper right, beginning with the lower-right diagonal. If the sum in a given diagonal is a two-digit number, then the tens digit of the sum is regrouped, or “carried,” to the next diagonal. **Figure 2b** shows an example using more digits in the factors. This method was probably first developed in India and made its way into Chinese, Arabian, and Persian work (Eves 1983b). The fourth graders and their teacher liked this method, which was also a favorite of the Arabs. This simple method might still be in use were it not for the need to draw the net of lines.

FIGURE 2

The lattice method



(a)
Will’s work in multiplying 237 × 7



(b)
The lattice method used to multiply 872 × 35

Another algorithm for multiplication, the doubling, or Egyptian, method, required me to introduce some information about the powers of 2 and the distributive property. To multiply 19 by 35 using the doubling method, write 19 as a sum of powers of 2, that is, $2^0 + 2^1 + 2^4$, or $1 + 2 + 16$. Starting with $1 \times 35 = 35$, find out what 2×35 is by doubling 35, that is, simply adding 35 to itself so that multiplication is not required. Continue in a similar manner until you reach 16×35, as follows:

$1 \times 35 = 35$
$2 \times 35 = 70$
$4 \times 35 = 140$ (obtained by adding 70 + 70)
$8 \times 35 = 280$ (obtained by adding 140 + 140)
$16 \times 35 = 560$ (obtained by adding 280 + 280)

Next add the partial products that correspond to the powers of 2 to make 19. Clearly, the distributive property tells us that $19 \times 35 = (1 + 2 + 16) \times 35 = 1 \times 35 + 2 \times 35 + 16 \times 35$. We must also understand, however, that we are solving 16×35 not by the standard multiplication algorithm but rather by the process of doubling. The need for this procedure is further highlighted if the students imagine doing the problem with Egyptian numerals. Students should appreciate that the decimal system, with its place-value characteristic, allows for an efficient multiplication algorithm. This exercise gives teachers the opportunity to review the standard algorithm for multiplication and to create a sense of the history of the mathematical achievements that have culminated in this wonderful tool!

By checking the answers obtained using the Egyptian method against those found using the standard algorithm, the students again verified that their results were correct. The lesson added a different dimension to what could have been another day of solving thirty multiplication problems on a worksheet in a traditional classroom. The students practiced multiplication, but they gained a multicultural perspective as well. Although I did not expect the fourth graders to remember the doubling method, I did seize the opportunity to make a connection with their learning of basic facts. If in the doubling algorithm, 8×35 is obtained by doubling the answer to 4×35, then students can apply the

FIGURE 3

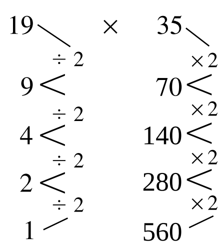
Spanish numerals and numeral names

1 uno	10 diez	19 diecinueve	60 sesenta
2 dos	11 once	20 veinte	70 setenta
3 tres	12 doce	21 veintiuno	80 ochenta
4 cuatro	13 trece	22 veintidos	90 noventa
5 cinco	14 catorce	30 treinta	100 ciento/cien
6 seis	15 quince	31 treinta y uno	200 dosceintos
7 siete	16 dieciseis	32 treinta y dos	300 trescientos
8 ocho	17 diecisiete	40 cuarenta	1000 mil
9 nueve	18 dieciocho	50 cincuenta	2000 dos mil

same reasoning to figure out answers from facts that they already know. For example, a student who knows that 2 × 7 is 14 can solve 4 × 7 by doubling 14. Such reasoning must be valued as students work toward the ability to recall facts automatically.

Duplation-and-Mediation Method, or Russian-Peasant Algorithm, for Multiplication

The final algorithm introduced to the fourth graders was the duplation-and-mediation method, or Russian-peasant algorithm. Because these fourth graders did not have the prerequisite knowledge of the division algorithm, I began this lesson by posing division questions, having the students model the problems with base-ten blocks, then showing them how to write the algorithm symbolically. To multiply 19 × 35 by duplation and mediation, successively divide 19 by 2, ignoring remainders. Successively double, that is, multiply by 2, the other factor, 35, for the same number of steps, as shown:



Select the odd numbers in the left column, and add the companion numbers in the right column. In this example, 19, 9, and 1 in the left column are odd numbers, so we add 35 + 70 + 560 to find the product of 19 and 35. The students were able to follow these steps and compare the results with those obtained by the standard algorithm. The same messages were reiterated: Here is another method that works, and some people actually use this method!

Other Numerals

From the beginning of our discussion of other numeration systems and algorithms, the students had asked about Japanese numerals. Part of their interest could have stemmed from one kindergarten teacher's recent trip to Japan as a Fulbright scholar, but their curiosity was also piqued by the presence of Japanese students at school. I followed up on the students' interest by showing them how to write Japanese numerals (Rowley 1992). They enjoyed the practice in writing these numerals.

Because the class had learned some Spanish, I also gave students a list of Spanish words for numerals (see **fig. 3**). As we practiced saying the words, we noted that the use of base ten was evident in the names for the numerals. I included a little multiplication practice by asking students to find the product of *nueve y siete*, for example. The students then had to figure out not only that 9 × 7 is 63 but also that 63 would be *sesenta y tres*. Again, some work on basic facts was included in the lesson, but the practice did not come in the monotonous form of a worksheet.

The final part of our project dealt with the words used for numerals. We discussed the fact that some cultures do not have written symbols for numbers. In some cultures, people point to certain body parts to signify specific numbers. The Kamayura tribe of South America, for example, uses the word for *peak finger* for *three* (Eves 1983a). The Malinke of West Sudan use the word *dibi*, which means *a mattress*, for *forty*. The explanation behind this choice is that two people share a mattress, and two people have a total of forty fingers and toes. Our own English words for numerals tend to be taken for granted. Children who have to memorize how to count in English have a harder time than children who speak some other languages, because the base of ten is not apparent in English. *Eleven* derives from *ein lifon*, meaning *one left over*, or *one over* (Eves 1983b, p. 4). *Twenty* comes from *twe-tig*, meaning *two 10s*.

Enhancing the Project in Your Classroom

To explore the meaning of larger numerals, ask students to write one billion as a standard numeral. Pretend to write a check for one billion dollars—what goes after the dollar mark? The expected answer is $1,000,000,000. Ask the students how many millions are in this amount. This question will help you determine whether they really understand place value. After the students give the expected answer that a billion is a thousand millions, ask them if that answer is always correct. Have them look *billion* up in a dictionary. They will find that a thousand millions is correct for the United States and France but that in Great Britain and Germany, a billion is a million millions. This exercise might also lead in to a discussion of the fact that some countries use commas for decimals and that the international system leaves spaces rather than commas between groups of three digits.

Interested teachers can find ways to develop many more of the kinds of ideas presented in this article. Two types of sources are particularly helpful. The first is books on the history of mathematics (e.g., Eves [1983b]; Newman [1956]; Ore [1948]). The other type of source is textbooks intended for use by preservice elementary school teachers (e.g., Bassarear [1997]; Bennett and Nelson [2000]; Billstein, Libeskind, and Lott [1997]; Long and DeTemple [1996]). Such resources can help you create activities to showcase mathematics as an avenue to awareness of other cultures and to enrich your students' mathematical journeys.

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